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INTERLEAVING BASED SCMA CODEBOOK DESIGN USING ARNOLD'S CAT CHAOTIC MAP

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***Abstract*- Non-orthogonal multiple access (NOMA) technology is a significant topic of the present 5G investigation. A possible NOMA approach is Sparse Code Multiple Access (SCMA), which increasesnext-generation communication's ability to handle multiple users. In SCMA, codebook design and optimization are crucial components that define the performance of multiple access systems. In this work, the complete design procedure of the SCMA codebook based on interleaving is explained and simulated. Furthermore, a chaotic interleaving based on ArnoldÃ¢sCat map is proposed to reduce the computational complexity. The performance of the SCMA codebook based on interleaving is evaluated through comparison with selected codebooks for SCMA multiplexing. The measures used for performance evaluation include bit error rate (BER), peak-to-average power ratio (PAPR), and minimum Euclidian distance (MED). The results show that the proposed codebook with chaotic interleaving achieves comparable performance to the traditional codebook based on interleaving, with less MED and higher BER compared to computer-generated and 16-star QAM codebook design methods but with reduced complexity.**

keywords: **MTM, 5G, Sub-6GHz, fractal, T-resonator**

I. Introduction

Numerous applications are supported by the cellular systems of the next generation, such as future factories, networked autonomous vehicles, smart homes and cities, electronic health, etc. [1]. As a result, there will be constant access to digital data services wherever and whenever they are needed. The rapid expansion in communication technologies and applications brings an increasingly congested spectrum, which is one of the major challenges in the design of 5G communication networks and beyond [2].

Multiple access, as one of the fundamental strategies used in wireless communication, enables several users to utilize the limited resources at once appropriately and effectively. Most traditional multiuser communication systems employ orthogonal multiple access (OMA) schemes in which different users have orthogonal relationships to a certain kind of resource. Every OMA system has an upper bound set by the number of orthogonal resources that can be used at once [1][2]. Non-orthogonal multiple access (NOMA) has received a lot of research interest recently, with the main goal of enabling huge connectivity, greater spectrum efficiency, and shorter communication latency compared to OMA. In the NOMA system, two or more users can share a single physical resource (e.g., power, frequency, time, or code) to provide a factor of overloading greater than one [3]. When compared to OMAsystems, NOMA has advantages, among them reduced transmission latency, great spectrum efficiency, and widespread connectivity [4].

Sparse Code Multiple Access (SCMA) is a non-orthogonal technique primarily based on the Multi-Dimensional (MD) codebook. SCMA is a brand-new non-orthogonal multiple access technique built on CDMA, wherein Quadrature Amplitude Modulation (QAM) symbols are dispersed over Orthogonal Frequency Division Multiple Access (OFDMA) tones and the

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Code Division Multiple Access (CDMA) encoder disperses these to create a complex symbol sequence that has been pre-designed [5]. Direct bit mapping into sparse codewords is achieved in SCMA by combining the mapping and spreading QAM symbol processes [6].

The pre-designed codebooks for these multidimensionally complex domains contain the codewords. Each user's codebook corresponds to a layer; therefore, a huge number of users will result in the existence of numerous multidimensional layers. As a result of the multidimensional modulation of the signals from the many users, the SCMA system benefits from coding gain, shaping gain, and code sparsity. Because a variety of detection approaches are being used, multiplexed codewords of low to moderate complexity can be decoded with the help of all these properties, which are advantageous to the receiver [5]. The SCMA system consists of several fundamental components, as shown in Fig. 1. Before the data is broadcast over the air and impacted by channel impairments, it is multiplexed into a signal and mapped into SCMA codewords during transmission.. After that, the base receptor receives them, where a decoder for the Message Passing Algorithm (MPA) is used to decode each user signal and recover the data delivered [5].

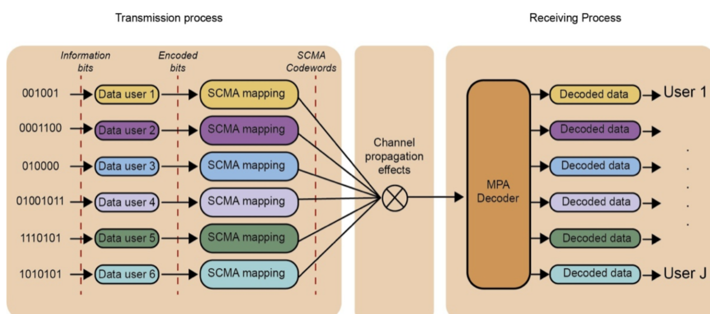

Figure 1: SCMA System Block Diagram [5]

II. RELATED WORKS

Many previous works in the literature are devoted to designing an efficient codebook for the SCMA system.
Caiet al. [8] created a codebook for the MD constellation optimization that employsrotation and interleaving. The method rotates a lattice that was initially designed to build multi-dimensions for making codebooks. Additionally, interleaving is used to enhance communication on fading channels. The performance results showed that the proposed method has a minor improvement compared to the Low-Density Signature (LDS) method. It achieves 10^{-4} BER at SNR equal to 15 dB SNR for a fading channel with reduced PAPR, while uplink resource allocation achieves 10^{-4} BER at 21 dB [5].

A similar strategy is employed by Bonilla [9]. It includes the interleaving and phase rotation with a Minimum Euclidian Distance (MED)-based detector at the receiver. It achieves $10^{-4}\mathrm{BER}$ for codebooks with 4 arrays, 8 arrays, and 16 arrays, respectively, Eb/ No equals $5\ \mathrm{dB}, 6\ \mathrm{dB}$, and 6.5 dB .

Liu [10], based on the factor graph, suggested segmentation and combining for MED improvement. Results indicate that irregular LDS performs pretty comparably to the suggested technique at greater SNR, which emphasizes the significance of an irregular factor graph of LDS. It achieves $10^{-4}\mathrm{BER}$ at 10 dB SNRoverfading channel with LDPC coding.

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Yu [11] offered a codebook design based on Star-QAM to enhance MED, The process includes creating four constellation point vectors that correspond to the components of an ndimensional star constellation. Mathematically, mapping vectors providesthe mother codebook.

Yasmine et al. [12] used Star-QAM constellations, and MED values were improved by using trellis coded modulation (TCM). Their method achieves 10^{-4} BER at 8 -10dB SNR over the AWGN channel. In this work, an SCMA codebook based on constellation rotation and interleaving is designed and simulated. To enhance the performance of the designed codebook, chaotic interleaving based on Arnold cat chaotic map which is the first time it is used with SCMA codebook design according to our knowledge.

III. SCMA CODEBOOK DESIGN

The MD constellation matrix is subjected to layer-specific operations Φ by the Codebook (CB) design., $\mathrm{CB}=\mathrm{MC}*\Phi$, where MC denotes the optimal MD constellation. Complex conjugate, phase rotation, and vector permutation are examples of the special operations used in the codebook design. Depending on the desired performance improvement in BER, the design may combine various operations. The mapping matrices of the factor graph are used to retrieve user codebooks from the mother codebook. The user codebooks are created by elementally multiplying each user's vector matrix by the codebook. Regarding K users, each with the codebook CB_{I}, The definition of the mother codebook matrix is $\mathrm{CB}=[\mathrm{CB}_1\mathrm{CB}_2\cdot\mathrm{CB}_K]$ [7].

The constellations can be optimized using a variety of methods for codebook design. The three subcategories of these techniques are factor graph optimization, constellation rotation with permutation, and latticebased optimization [8]. In this work, multidimensional SCMA codebook design is applied based on constellation rotation and proposed interleaving based on using Chaotic Arnold Cat's map [13]. The steps of MC generation and constellation rotation can be explained by the following steps:

A. First, the mother constellation can be defined, which is each point's representation of the constellation [4][8]. S_1 is a definition of a subset of z^2, where z is anintegers, is given by (1):

$$S_{1m}=A_m(1+j)|_{A_m=2m-1-M}\cdot\quad m=1.2\ldots\ldots..M \tag{1}$$

Where M is referred to as the code word number.

B. values of the first-dimension constellation, with $M=4$, each $S1$ point is given a value. The form of Gray's mapping is [4][8]:

$$\mathrm{S}_{11}00\longrightarrow-3(1+\mathrm{i})$$

$$\mathrm{S}_{12}01\longrightarrow-(1+\mathrm{i})$$

$$\mathrm{S}_{13}11\longrightarrow(1+\mathrm{i})$$

$$\mathrm{S}_{14}10\longrightarrow3(1+\mathrm{i})$$

S_{11}	S_{12}	S_{13}	S_{14}
$-3(1+\mathrm{i})$	$-(1+\mathrm{i})$	$(1+\mathrm{i})$	$3(1+\mathrm{i})$
00	01	11	10

C. We next give θ_{l-1}, the factor that flips the vector S 1 to produce the multi-dimension in the MC. Fig. 2 displays the MC utilizing the base vector S 1 and the N dimensions after rotating the vector, which is described as [4][8]:

$$\theta_{\mathrm{l}-1} = (\mathrm{l}-1) \times \frac{\pi}{\mathrm{MN}}. \quad \mathrm{l} = 1.2\ldots\ldots\ \mathrm{N} \tag{2}$$

Setting $N = 2$ and $M = 4$ causes the angle of rotation phases in the MC to be calculated as:

$$\text{For l} = 1, \text{ we have } \theta_{l-1} = \theta_0 = (1-1) \times \frac{\pi}{(4)(2)} = 0$$

$$\text{For l} = 2, \text{ we have } \theta_{l-1} = \theta_1 = (2-1) \times \frac{\pi}{(4)(2)} = \frac{\pi}{8}$$

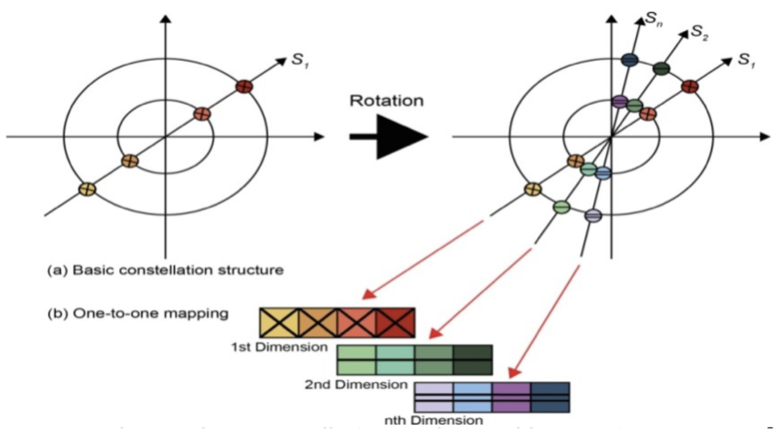

Figure 2: The Mother Constellation Is Obtained by Rotating Vector S1 [8]

D. The dimensions of the MC are determined by the angles, which rotate the original vector. $\mathrm{S_N}$ originates from and is defined as $S_N = S_1 U_N$, where UN is a multidimensional phase rotation matrix having the following definitions:

$$U_N = \operatorname{diag}\left(1e^{1\theta_{l-1}}\right) \subset \mathrm{C}^{\mathrm{N}\times\mathrm{M}} \tag{3}$$

When $\theta_0 = 0$, we have:

$$\mathrm{s}_{11}00 \longrightarrow -3(1+\mathrm{i})$$

$$\mathrm{s}_{12}01 \longrightarrow -(1+\mathrm{i})$$

$$\mathrm{s}_{13}11 \longrightarrow (1+\mathrm{i})$$

$$\mathrm{s}_{14}10 \longrightarrow 3(1+\mathrm{i})$$

and when $\theta_1 = \frac{\pi}{8}$, we have:

$\mathrm{S}_{21} = -1.62 - 3.92\mathrm{i}$

$\mathrm{S}_{22} = -0.54 - 1.31\mathrm{i}$

$\mathrm{S}_{23} = 0.54 + 1.31\mathrm{i}$

$\mathrm{S}_{24} = 1.62 + 3.92\mathrm{i}$

Fig. 3 displays the main constellation.

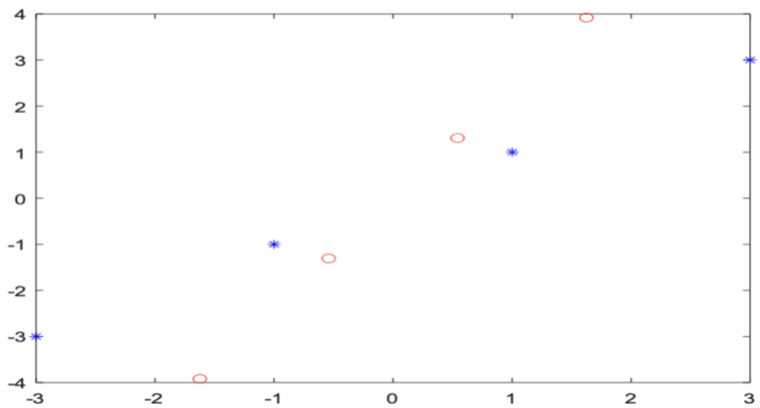

Figure 3: Mother Constellation and Rotation Of the Constellation Of Eq.(4)

E. After the S vectors have been obtained, the interleaving of the even-numbered S vectors is completed. S 2 is symbolized by Se :

$$\mathrm{S}_{e1} = \mathrm{S}_{21} = -3(1+\mathrm{i})\mathrm{e}^{\mathrm{i}\pi/8}$$

$$\mathrm{S}_{e2} = \mathrm{S}_{22} = -(1+\mathrm{i})\mathrm{e}^{\mathrm{i}\pi/8}$$

$$\mathrm{S}_{e3} = \mathrm{S}_{23} = (1+\mathrm{i})\mathrm{e}^{\mathrm{i}\pi/8}$$

$$\mathrm{S}_{e4} = \mathrm{S}_{24} = 3(1+\mathrm{i})\mathrm{e}^{\mathrm{i}\pi/8}$$

As in [5] and [8], for interleaving, only even dimensions (rows) of the MC are reordered. For example, after interleaving, the Se vector is se^*, where e is an even number in N and is expressed as:

$S_e^* = \{-S_e, M/2+1, \ldots, -S_e, 3M/4, \text{Se }, 3M/4+1, \ldots, -S_e, M, Se, M, \ldots, -S_e, 3M/4, S_e, 3M/4, \ldots, S_e, M/2+1\}$

We propose to write Eq.(5) in an easier way as:

$$S_e^* = \left\{ -\mathrm{S}_e \cdot \mathrm{Rem}\left(\frac{\frac{M}{2}}{M+1}\right) \ldots + \mathrm{S}_e \cdot \mathrm{Rem}\left(\frac{M}{M+1}\right) \cdot -\mathrm{S}_e \cdot \mathrm{Rem}\left(\frac{4M}{3(M+1)}\right) \ldots . + \mathrm{S}_e \cdot \mathrm{Rem}\left(\frac{2M}{M+1}\right) \right\} \quad (5)$$

For $M = 4$, the interleaved vector then becomes

$$S_2 = \begin{bmatrix} -0.54 - 1.31i & 1.62 + 3.92i & -1.62 - 3.92i & 0.54 + 1.31i \end{bmatrix}$$

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F. After the $\mathrm{S_N}$ vector has been rotated, it is possible to write the N -dimensional matrix MC with M points as:

$$MC = (S_1, S_2, \ldots S_N)^T = \begin{bmatrix} \mathrm{S}_{11} & \mathrm{S}_{12} \ldots & \mathrm{S}_{1N} \\ \vdots & \ddots & \vdots \\ \mathrm{S}_{N1} & \ldots & \mathrm{S}_{MN} \end{bmatrix} \tag{6}$$

when $\mathrm{N} = 2$, the resulting MC is:

$$MC = \begin{bmatrix} -3.00 - 3.00\mathrm{i} & -1.00 - 1.00\mathrm{i} & 1.00 + 1.00\mathrm{i} & 3.00 + 3.00\mathrm{i} \\ -0.54 - 1.31\mathrm{i} & 1.62 + 3.92\mathrm{i} & -1.62 - 3.92\mathrm{i} & 0.54 + 1.31\mathrm{i} \end{bmatrix}$$

IV. THE PROPOSED METHOD FOR INTERLEAVING

Arnold's cat map is a square matrix expressed as [14]

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \bmod N \tag{7}$$

Where N is the maximum index of the two-dimentional matrix (i.e, the $\mathrm{N} \times \mathrm{N}$ elements matrix) to be interleaved using Arnold's cat map. The coordinates of each element that the ordered pair represents (X, Y) are in the range $[0, 1]$ when modulo 2 is applied. It is mainly used for interleaving the pixels in image processing applications. It creates a discrete-time dynamical system in which the mapping Γ_{cat} ' iterations control the evolution as follows:

$$\Gamma_{\mathrm{cat}} = \begin{bmatrix} X_{n+1} \\ Y_{n+1} \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} X_n \\ Y_n \end{bmatrix} \bmod N \tag{8}$$

Where $\mathrm{N} = \sqrt{M}$. To improve SCMA performance and reduce the computational complexity, we propose using the Arnold cat map for interleaving the mother constellation using Eq.(8) instead of the interleaving process expressed by Eq.(4). To perform the interleaving of vector S_2 using Arnold's cat map, S_2 should be converted into a $\mathrm{N}X$ N square matrix and processed. When N is 2, S_2 would be a 2×2 matrix as follows:

$$\mathrm{S}_2 = \begin{bmatrix} -0.54 - 1.31\mathrm{i} & -1.62 - 3.92\mathrm{i} \\ 1.62 + 3.92\mathrm{i} & 0.54 + 1.31\mathrm{i} \end{bmatrix}$$

then the locations of each element in the S_2 matrix would be changed using the Arnold cat map as in Eq.(8), where $\begin{bmatrix} X_n \\ Y_n \end{bmatrix}_{S_2}$ is the location of elements before interleaving, and $\begin{bmatrix} X_{n+1} \\ Y_{n+1} \end{bmatrix}$ is the corresponding locations after interleaving. Hence, S_2 after interleaving would be:

$$\mathrm{S}_{21} = 1.6236 + 3.9197\mathrm{i}$$

$$\mathrm{S}_{22} = -1.6236 - 3.9197\mathrm{i}$$

$$\mathrm{S}_{23} = 0.5412 + 1.3066\mathrm{i}$$

$$\mathrm{S}_{24} = -0.5412 - 1.3066\mathrm{i}$$

As a result, MC with chaotic Arnold map interleaving would be as follows:

$$MC=\begin{bmatrix} -3.00-3.00\mathrm{i} & -1.00-1.00\mathrm{i} & 1.00+1.00\mathrm{i} & 3.00+3.00\mathrm{i} \\ 1.62+3.92\mathrm{i} & -1.62-3.92\mathrm{i} & 0.54+1.31i & -0.54-1.31i \end{bmatrix}$$

The main constellation and rotation using the proposed chaotic interleaving can be explained using the flowchart shown in Fig. 4.

V. GENERATION OF SPARSE CODEWORDS

After the complex Mother Constellation is obtained, they are added to the sparse codewords. The codewords for the various symbols transmitted by user j are concatenated to create a codebook X_j for that single user. In the codebook, each symbol is represented by a column vector with K rows. Two of the rows have complex numbers in them, and the other two have the value 0 . Consequently, the SCMA codebook X_j, often known as for the jth user, is generated as follows [4][5] [8]:

$$\mathrm{X_j}=\mathrm{V_j}\Delta_\mathrm{j}\mathrm{MC}. \quad \mathrm{j}=1.2\ldots\ldots.\mathrm{J} \tag{9}$$

A sparsely populated $\mathrm{V_j}$ represents the dispersion matrix for each user, where $\mathrm{V}\in B^{K\times N}$, mapping the K -dimensional codeword X onto the N -dimensional complex constellation point. In this state, if $\mathrm{K}=4,\ \mathrm{N}=2$ and $\mathrm{J}=6$, the matrices $V_j, j=1,2,\ldots J$ are:

$$\mathrm{V}_1=\begin{bmatrix}0&0\\1&0\\0&0\\0&1\end{bmatrix}\quad \mathrm{V}_2=\begin{bmatrix}1&0\\0&0\\0&1\\0&0\end{bmatrix}\quad \mathrm{V}_3=\begin{bmatrix}1&0\\0&1\\0&0\\0&0\end{bmatrix}$$

$$\mathrm{V}_4=\begin{bmatrix}0&0\\0&0\\1&0\\0&1\end{bmatrix}\quad \mathrm{V}_5=\begin{bmatrix}1&0\\0&0\\0&0\\0&1\end{bmatrix}\quad \mathrm{V}_6=\begin{bmatrix}0&0\\1&0\\0&1\\0&0\end{bmatrix}$$

Rotating the mother constellation is necessary for the codebook design for different users. The rotation of angles phase can be formulated as follows:

$$\varphi_u=(u-1)\frac{2\pi}{\mathrm{Mdf}}+e_u\frac{2\pi}{\mathrm{M}}. \quad \forall \mathrm{u}=1,\ldots,\mathrm{df} \tag{10}$$

where e_u is an integer representation of Z and df represents the overloading factor, or the number of elements in a given subcarrier at once. φ_u are the minimum ideal rotation phase angles that maintain the distance between the user's dimensions on the df interfering layers.

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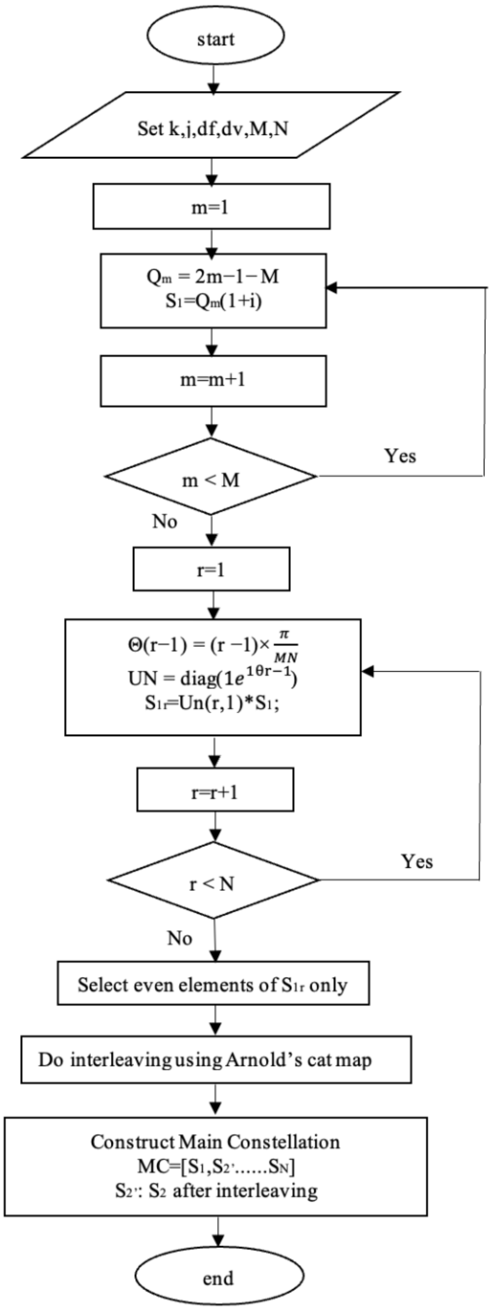

Figure 4: Flow Chart Of the Main Constellation and Rotation Using the Proposed Chaotic Interleaving

The computed values from Eq.(10) if $M = 4$ and $df = 3$ are: $\varphi_1 = 0, \varphi_2 = \frac{\pi}{6}$ and $\varphi_3 = \frac{\pi}{3}$. These are ideal values dependent on Latin squares that keep the consistent Euclidean distance between codewords and the structure of the φ_1 mother codebook [5][8]. The rotation angle factor graph that follows this principle is:

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$$F_{\varphi}=\begin{bmatrix} \varphi 1 & 0 & \varphi 2 & 0 & \varphi 3 & 0 \\ \varphi 2 & 0 & 0 & \varphi 3 & 0 & \varphi 1 \\ 0 & \varphi 2 & \varphi 1 & 0 & 0 & \varphi 3 \\ 0 & \varphi 3 & 0 & \varphi 1 & \varphi 2 & 0 \end{bmatrix} \tag{11}$$

According to the codebook's structure, as long as it maintains the minimum dimensional distance between the layers, there are numerous ways to arrange the phase rotation [5][8]. The angles are condensed in a matrix $F_{\varphi j}$ as:

$$F_{\varphi j}=\begin{bmatrix} \varphi 1 & \varphi 2 & \varphi 2 & \varphi 3 & \varphi 3 & \varphi 1 \\ \varphi 2 & \varphi 3 & \varphi 1 & \varphi 1 & \varphi 2 & \varphi 3 \end{bmatrix} \tag{12}$$

from the condensed angles of $F_{\varphi j}$ the rotation operator Δ_{j} is produced as follows:

$$\Delta_j=\operatorname{diag}\left(F_{\varphi j}\right), \quad \forall \mathrm{j}=1,2,\ldots, \mathrm{~J} \tag{13}$$

In this case when $\mathrm{J}=6$, we have:

$$\Delta 1=\begin{bmatrix} \varphi 1 & 0 \\ 0 & \varphi 2 \end{bmatrix} \quad \Delta 2=\begin{bmatrix} \varphi 2 & 0 \\ 0 & \varphi 3 \end{bmatrix} \Delta 3=\begin{bmatrix} \varphi 2 & 0 \\ 0 & \varphi 1 \end{bmatrix}$$

$$\Delta 4=\begin{bmatrix} \varphi 3 & 0 \\ 0 & \varphi 1 \end{bmatrix} \Delta 5=\begin{bmatrix} \varphi 3 & 0 \\ 0 & \varphi 2 \end{bmatrix} \Delta 6=\begin{bmatrix} \varphi 1 & 0 \\ 0 & \varphi 3 \end{bmatrix}$$

So, the SCMA codebooks X_j for the jth user using eq. (8) and after normalizing the values are:

$$\mathrm{X}_1=\begin{bmatrix} 0.0000+0.0000\mathrm{i} & 0.0000+0.0000\mathrm{i} & 0.0000+0.0000\mathrm{i} & 0.0000+0.0000\mathrm{i} \\ -0.2481-0.2481\mathrm{i} & -0.0827-0.0827\mathrm{i} & 0.0827+0.0827\mathrm{i} & 0.2481+0.2481\mathrm{i} \\ 0.0000+0.0000\mathrm{i} & 0.0000+0.0000\mathrm{i} & 0.0000+0.0000\mathrm{i} & 0.0000+0.0000\mathrm{i} \\ 0.3827+0.9239\mathrm{i} & -0.3827-0.9239\mathrm{i} & 0.1276+0.3080\mathrm{i} & -0.1276-0.3080\mathrm{i} \end{bmatrix}$$

$$\mathrm{X}_2=\begin{bmatrix} -0.2481-0.2481\mathrm{i} & -0.0827-0.0827\mathrm{i} & 0.0827+0.0827\mathrm{i} & 0.2481+0.2481\mathrm{i} \\ 0.0000+0.0000\mathrm{i} & 0.0000+0.0000\mathrm{i} & 0.0000+0.0000\mathrm{i} & 0.0000+0.0000\mathrm{i} \\ 0.3827+0.9239\mathrm{i} & -0.3827-0.9239\mathrm{i} & 0.1276+0.3080\mathrm{i} & -0.1276-0.3080\mathrm{i} \\ 0.0000+0.0000\mathrm{i} & 0.0000+0.0000\mathrm{i} & 0.0000+0.0000\mathrm{i} & 0.0000+0.0000\mathrm{i} \end{bmatrix}$$

$$\mathrm{X}_3=\begin{bmatrix} -0.7071-0.7071\mathrm{i} & -0.2357-0.2357\mathrm{i} & 0.2357+0.2357\mathrm{i} & 0.7071+0.7071\mathrm{i} \\ 0.1343+0.3242\mathrm{i} & -0.1343-0.3242\mathrm{i} & 0.0448+0.1081\mathrm{i} & -0.0448-0.1081\mathrm{i} \\ 0.0000+0.0000\mathrm{i} & 0.0000+0.0000\mathrm{i} & 0.0000+0.0000\mathrm{i} & 0.0000+0.0000\mathrm{i} \\ 0.0000+0.0000\mathrm{i} & 0.0000+0.0000\mathrm{i} & 0.0000+0.0000\mathrm{i} & 0.0000+0.0000\mathrm{i} \end{bmatrix}$$

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$$X_4 = \begin{bmatrix} 0.0000 + 0.0000i & 0.0000 + 0.0000i & 0.0000 + 0.0000i & 0.0000 + 0.0000i \\ 0.0000 + 0.0000i & 0.0000 + 0.0000i & 0.0000 + 0.0000i & 0.0000 + 0.0000i \\ -0.7071 - 0.7071i & -0.2357 - 0.2357i & 0.2357 + 0.2357i & 0.7071 + 0.7071i \\ 0.0471 + 0.1138i & -0.0471 - 0.1138i & 0.0157 + 0.0379i & -0.0157 - 0.0379i \end{bmatrix}$$

$$X_5 = \begin{bmatrix} -0.7071 - 0.7071i & -0.2357 - 0.2357i & 0.2357 + 0.2357i & 0.7071 + 0.7071i \\ 0.0000 + 0.0000i & 0.0000 + 0.0000i & 0.0000 + 0.0000i & 0.0000 + 0.0000i \\ 0.0000 + 0.0000i & 0.0000 + 0.0000i & 0.0000 + 0.0000i & 0.0000 + 0.0000i \\ -0.1343 - 0.3242i & 0.1343 + 0.3242i & 0.0448 + 0.1081i & -0.0448 - 0.1081i \end{bmatrix}$$

$$X_6 = \begin{bmatrix} 0.0000 + 0.0000i & 0.0000 + 0.0000i & 0.0000 + 0.0000i & 0.0000 + 0.0000i \\ -0.0871 - 0.0871i & -0.0290 - 0.0290i & 0.0290 + 0.0290i & 0.0871 + 0.0871i \\ 0.3827 + 0.9239i & -0.3827 - 0.9239i & 0.1276 + 0.3080i & -0.1276 - 0.3080i \\ 0.0000 + 0.0000i & 0.0000 + 0.0000i & 0.0000 + 0.0000i & 0.0000 + 0.0000i \end{bmatrix}$$

Fig. 5 shows the codebook and codewords of the first user. The codebook for each user from the main constellation can be presented in the flow chart as shown in Fig. 6.

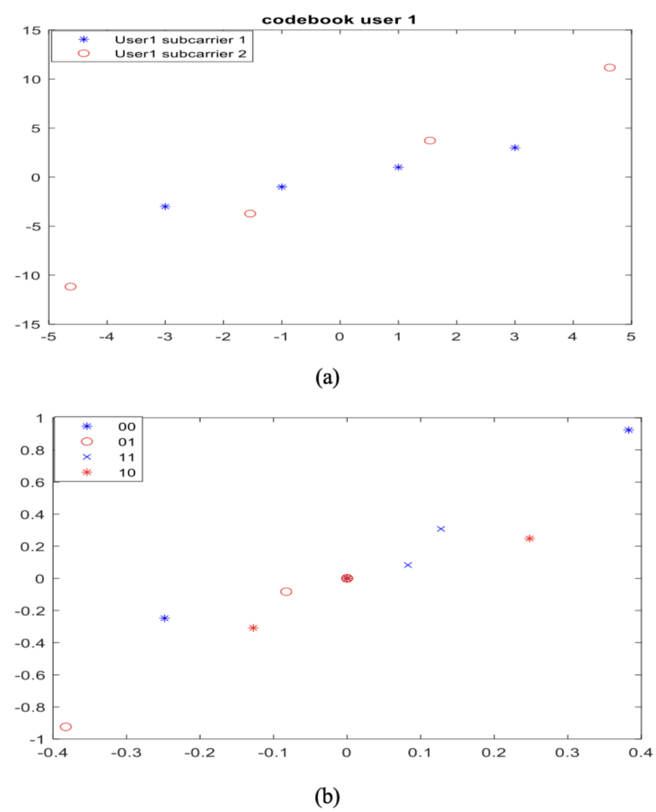

Figure 5: First User Codebook and Codewords. (a) Codebook, (b) Codewords

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VI. SIMULATION RESULTS

This section presents the results obtained from MATLAB simulations of the designed chaotic interleaving-based SCMA codebooks. The performance of the designed SCMA systems is measured in terms of MED, PAPR, computational complexity, and BER over the AWGN channel.

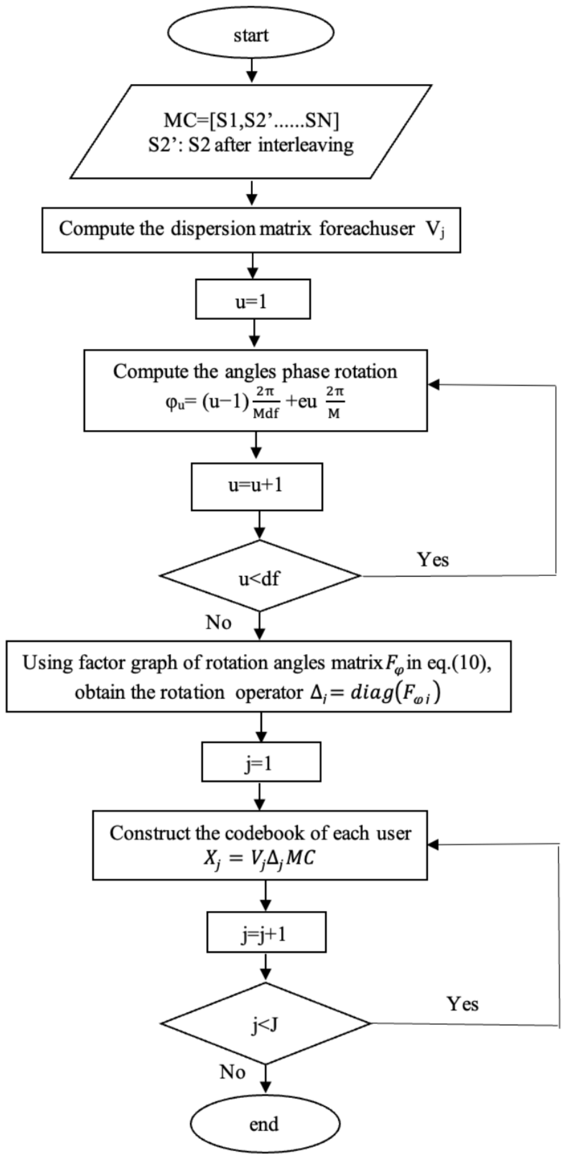

Figure 6: Flow Chart Of Codebook Design for Each User From the Main Constellation

The minimum square Euclidean distance of the codeword of the mother codebook is computed using the formula given in [8]:

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$$d_{\min}^2 = \begin{cases} 8\,[\ \mathrm{N}_0^{\sim} + 4\ \mathrm{N}_e^{\sim}] & \mathrm{M} = 4 \\ 8\ \mathrm{N} & \mathrm{M} > 4 \end{cases} \tag{14}$$

where $\mathrm{N}_0^{\sim}$ and $\mathrm{N}_e^{\sim}$ are the numbers of odd and even dimensions, respectively. On the other hand, PAPR is computed using [8]:

$$\mathrm{PAPR(MC)} = 10\log_{10}\left(\frac{3\left[No(M-1)^2 + Ne\right]}{N\left(M^2-1\right)}\right) \tag{15}$$

To evaluate the performance of the proposed chaotic interleaving-based SCMA codebook, it was compared with other methods in the literature. These methods include [5], [8], [10], and [12]. The methods in [5] and [8] were designed as codebooks with maximized minimum Euclidean distance (MinED) through an interleaving scheme described in Section V. The codebook in [10] for uplink wireless communications is provided by segmenting an optimal 16-point QAM into various subsets with the aim of minimizing MinED and the collisions of the information bits on the resources. The codebook design method in[12], Computer Generated Sparse Code Multiple Access (CG-SCMA), generates a complex SCMA codebook that maximizes MinED of 16-QAM. This codebook is generated using a computer program to specify the most appropriate values for this constellation, followed by TCM, which is used to divide the constellation into four subconstellations to maximize MinED.

Table I shows the MED and PAPR of the proposed SCMA as compared with some works available in the literature, while Fig. 7 and Fig. 8 show the BER comparison of the proposed chaotic interleaving method with the methods in [8], [10], and [12] for $M = 4, N = 2, J = 6$, and $k = 4$. It can be seen from these results that MED is maximized when $M = 4$, and the proposed method achieves almost the same performance as works in [5] and [8] in terms of MED, PAPR, and obtainable SNR at $\mathrm{BER} = 10^{-4}$. The results also show that 16 -star QAM and CG-SCMA methods achieve a better BER than other methods due to the coding associated with modulation.

The other important performance measure that should be considered in codebook design is computational complexity. This is because it requires less memory storage, which increases the operational speed at both encoding and decoding operations. The proposed interleaving method has low complexity as compared to other methods since it requires a smaller number of mathematical operations, as will be explained.

TABLE I
Comparison Of Different SCMA Methods

Method	J	K	M	N	*df*	SNR at BER=10-4	PAPR	MED
MDconst[5][8]	6	4	4	2	3	21 dB	0	2
Proposed	6	4	4	2	3	22 dB	0	2
MDconst[5][8]	8	6	4	3	4	23 dB	1.9629	1.64
Proposed	8	6	4	3	4	24 dB	1.9629	1.64
MDconst[5][8]	6	4	16	2	3	27 dB	0	0.88
Proposed	6	4	16	2	3	28 dB	0	0.88
16 starqam[10]	6	4	4	2	3	10 dB	0	2.11
CG SCMA[12]	6	4	4	2	3	$8-9$ dB	0	2.16

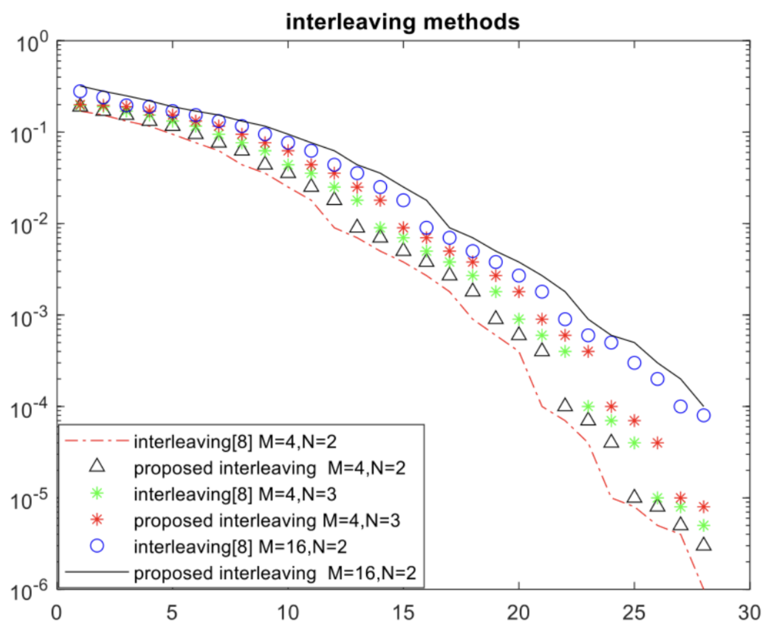

Figure 7: BER Comparison Of Proposed Chaotic Interleaving and Interleaving Scheme in [8]

By inspecting Eq.(5), it can be easily seen that 11 mathematical operations (division and multiplication) are required to perform interleaving for $M = 4$, while 23 operations are required for $M = 8$. As a result, the number of mathematical operations required in [5] and [8] for the interleaving method equals $3M - 1$. In contrast, the proposed interleaving method in Eq.(8) requires 8 mathematical operations for $M = 4$ and 16 for $M = 8$. As a result, the proposed interleaving method requires 2 M mathematical operations. Therefore, the percentage of reduction in the mathematical operations can be expressed as:

$$\% \text{ Reduction in operations } = \frac{3M-1-2M}{3M-1} = \frac{M-1}{3M-1} \quad (16)$$

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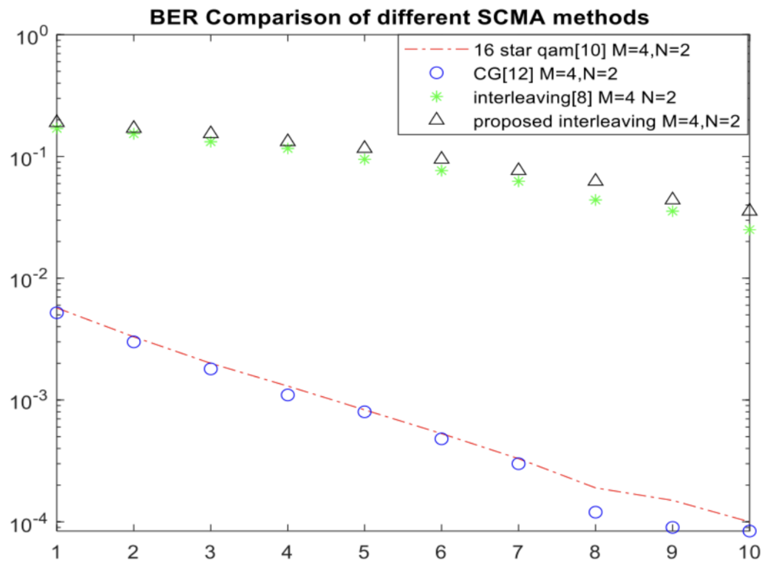

Figure 8: BER Comparison of Different SCMA Methods

The numerical comparison of computation complexity between the interleaving method in [5] and [8] and the proposed method is shown in Table II and Fig. 9. The obtained results indicate that the reduction in computational complexity using the proposed chaotic interleaving increases as M increases.

TABLE II
Comparison of Different SCMA Interleaving Methods in Complexity

M (number of code words)	Number of mathematical operations (multiplication and division)		Reduction in mathematical operations
	Interleaving method in [5] and [8]	Proposed chaotic interleaving	
4	11	8	27%
8	23	16	30%
16	47	32	32%

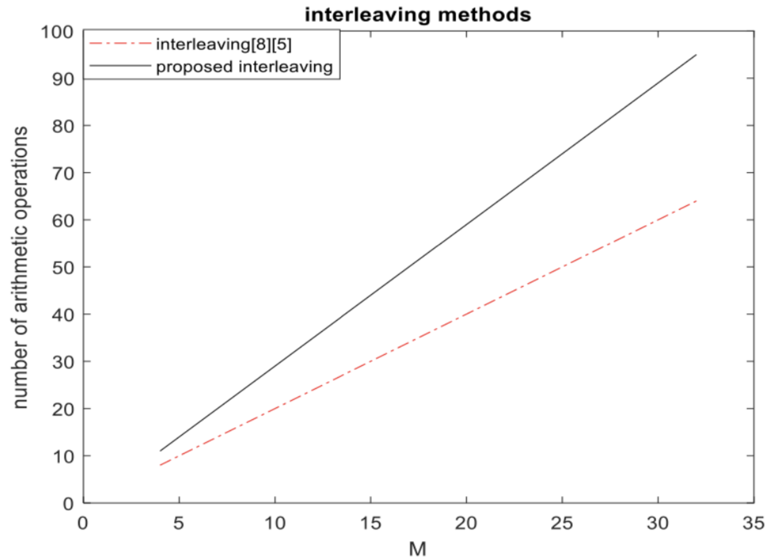

Figure 9: Complexity Comparison Of Proposed Interleaving Scheme and Schemes in [5] and [8]

CONCLUSION

The methods described for creating and improving the codebooks aim to reduce BERand PAPR and maximize the minimum Euclidian distance. The performance evaluation and comparison between different methods depends on the number of users that share the same resource and the number of available resources. The proposed codebook based on interleaving using Arnoldas cat map provides comparable performance to existing methods based on interleaving with reduced computational complexity.

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CONFLICTS OF INTEREST

The author declares no conflict of interest.

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